IOP Publishing European Journal of Physics
Eur. J. Phys. **29** (2008) 251–256 doi:10.1088/0143-0807/29/2/007

Studying charged particle optics: an undergraduate course

V Ovalle[1], D R Otomar[1], J M Pereira[2], N Ferreira[1], R R Pinho[3] and A C F Santos[2,4]

[1] Instituto de Fisica, Universidade Federal Fluminense, Av. Gal. Milton Tavares de Souza s/n°. Gragoatá, Niterói, 24210-346 Rio de Janeiro, Brazil
[2] Instituto de Fisica, Universidade Federal do Rio de Janeiro, Caixa Postal 68528, Rio de Janeiro, Brazil
[3] Departamento de Física–ICE, Universidade Federal de Juiz de Fora, Campus Universitário, 36036-900, Juiz de Fora, MG, Brazil

E-mail: toni@if.ufrj.br (A C F Santos)

Received 23 August 2007, in final form 12 December 2007
Published 17 January 2008
Online at stacks.iop.org/EJP/29/251

Abstract
This paper describes some computer-based activities to bring the study of charged particle optics to undergraduate students, to be performed as a part of a one-semester accelerator-based experimental course. The computational simulations were carried out using the commercially available SIMION program. The performance parameters, such as the focal length and *P*–*Q* curves are obtained. The three-electrode einzel lens is exemplified here as a study case.

Introduction

For many decades, physicists have been employing charged particle beams in order to investigate elementary processes in nuclear, atomic and particle physics using accelerators. In addition, many areas such as biology, chemistry, engineering, medicine, etc, have benefited from using beams of ions or electrons so that their phenomenological aspects can be understood. Among all its applications, electron microscopy may be one of the most important practical applications of lenses for charged particles. Most of the theoretical backgrounds of electrostatic and magnetic lenses were worked out by the pioneers and inventors of electron microscopy [1–3].

Transporting and controlling beams of charged particles are done, in most cases, by using lenses or a group of lenses. They are found to be very important in accelerating charged particles in the accelerator in which transporting and controlling are essential for the beams impinging on the target. As is well known from geometrical optics, the lenses have a focusing

[4] Author to whom any correspondence should be addressed.

action that maximizes the transmission. In the context of charged particles, the lenses are made of electrostatics or magnetic fields or a combination of both. They can be of several different types. The two most common types are the cylindrical and the circular apertures.

The aim of this paper is to describe, by using the computer program SIMION [4], some examples of class assignments about the behaviour of quantities concerned with electrostatic lenses and their focal properties. The choice of the SIMION software, was made because, as far the authors are concerned, it is widely used, user friendly and has speed and accuracy to enable it to backup many veridical coverings. Alternatively, the simulations could be run in other software, for instance LENSYS [5], which also uses the finite difference method, or the CPO program [6], which is based on the boundary element method. For a comparison of the performance of different software the reader is referred to [7]. For this purpose, an electrostatic cylindrical lens, the so-called einzel lens, which consists of three electrodes was used as a case study. The tasks are suitable for undergraduates with some background in electrostatics.

Background

The magnitudes of the energies of particles dealt with in this paper are low enough, so that relativistic effects can be ignored. Charged particles with equal kinetic energies, travelling inside purely electrostatic fields will follow the same trajectories, independent of their mass or charge, due to the fact that the differential equations that govern the motion of particles do not include neither their mass nor their charge. Thus, electrons, positrons, negatively and positively charged ions, will always have the same path.

An electrostatic lens consists of an array of electrodes held at different potentials. Although, there are appreciable similarities to optical lenses, electrostatic lenses have important differences, such as larger aberrations and the repulsive Coulomb force among the beam particles.

The physics of a charged particle beam penetrating a region of an electric field is similar to the situation in light optics. When a light ray moves through one medium to another, the refractive index modifies. In the case of a charged particle beam, the particles will be accelerated or decelerated, and the trajectory will depend on the angle of incidence with respect to the equipotential surfaces of the field. There is an analogue of Snell's law for charged-particle optics

$$\sqrt{V_1}\sin\alpha_1 = \sqrt{V_2}\sin\alpha_2, \qquad (1)$$

where V_1 and V_2 are the potentials referenced to the particle source, and α_1 and α_2 are the angles of incidence and refraction with respect to the normal of equipotential surfaces [8].

A comparison of equation (1) with Snell's law, allows us to see that the analogue of the refraction index is the square root of particle energy, or proportional to the particle velocity. As opposed to light optics, where the refractive index changes abruptly in the surface separating two media, the change in velocity of a beam of charged particles varies continuously across an electrostatic lens. This is the main difference between light and charged particle optics. Thus, an electrostatic lens always works as a thick lens. The focal points of a thick lens are situated at the focal lengths, namely, f_1 and f_2, measured from principal planes H_1 and H_2. Whereas the focal points with respect to a central plane, called M are F_1 and F_2. Figure 1 shows the focal point F_2 for the case of a symmetrical lens. The focal point F_1, in the case of a symmetrical lens is located symmetrically to F_2.

The positions of the principal planes, the focal points F_1 and F_2, object and image distances, P and Q, respectively, are measured with respect to the reference plane M

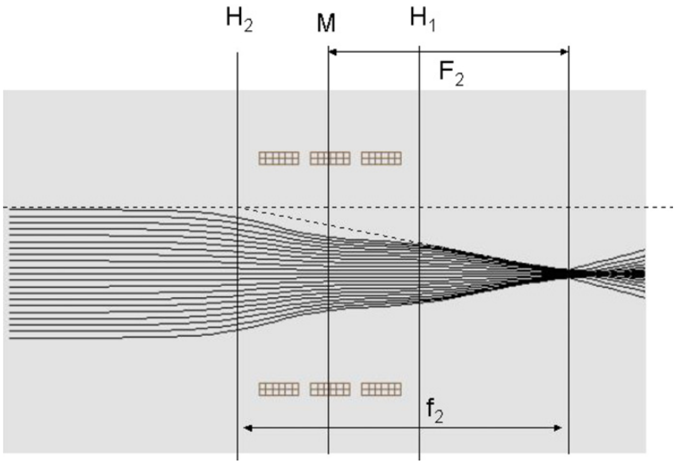

Figure 1. A parallel beam of electrons being focused by a symmetric three-element lens. The figure shows some of the lens parameters (see text for details).

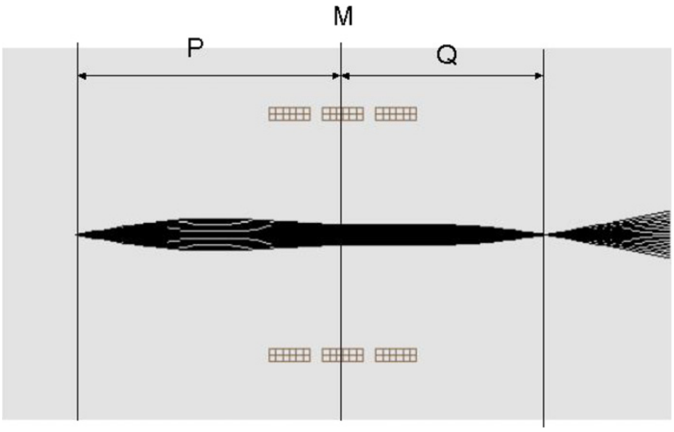

Figure 2. Lens parameters. P and Q are the object and image positions, respectively, with respect to the central plane M, which is chosen as being the mechanical symmetry plane of the lens.

that is often chosen as being the mechanical symmetry plane of the lens as shown in figures 1 and 2. For a detailed description of the fundamentals of geometrical optics, the reader is referred to [8].

For constructing an image from an object it is necessary to only have two rays. The details of the particle's trajectories within a lens do not need to be taken into account. Instead, only the asymptotic trajectories are considered. A ray parallel to the optical axis follows a straight line until the principal plane H_2 in which it suffers an abrupt refraction leaving it and passing through the focal point F_2, as shown by the dotted lines in figure 1. A ray passing through the focal point F_1 follows a straight line until the principal plane H_1 where it suffers an abrupt refraction leaving it and going on parallel to the optical axis.

The points of intersection of the first and second rays give the location of the image point corresponding to the object point. The lens data are presented commonly in the form P versus Q curves, where P and Q are scaled to the lens diameter D as shown in figure 3. The curves

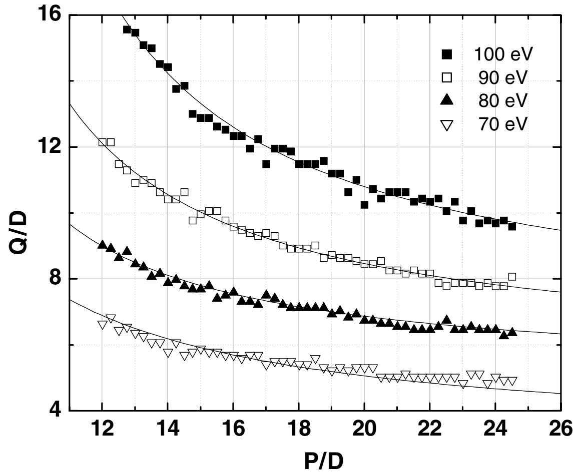

Figure 3. *P–Q* curves for a three-cylinder symmetrical lens for different electron energies. The full lines represent plots of equation (2).

are hyperbolas, each of them corresponding to a particular value of V_2/V_1. A great number of useful relationships can be derived geometrically with respective parameters given in figure 1, for instance the Newtonian lens equation:

$$(P - F_1)(Q - F_2) = f_1 f_2 \tag{2}$$

and

$$\frac{f_1}{f_2} = \sqrt{\frac{V_1}{V_2}}. \tag{3}$$

In the case of a weak lens (large f), the principal planes are close to the central plane. Then,

$$p \to P \qquad q \to Q \qquad f_1 \approx f_2 \approx f. \tag{4}$$

And equation (2) reduces to the well-known formula of geometrical optics for a thin lens

$$\frac{1}{P} + \frac{1}{Q} = \frac{1}{f}. \tag{5}$$

The cylinderical lenses are the most common types of lens. The length of each cylinder should be large compared with its diameter so that the axial potential can reach its asymptotic value. The focal parameters of the lenses, f_1, f_2, F_1 and F_2, depend upon the diameters of the cylinders, the spacing between them and the ratio V_2/V_1 applied to the two electrodes.

Some examples of simulations

By using the SIMION program [4], the students are asked to design and model the focal properties of an electrostatic lens. The main advantage of using this software is that the modelling can be easily performed avoiding analytical solutions. In this section, we shall describe some results using a symmetrical three-element einzel lens.

Figure 1 displays the geometry of a three-element lens comprising three coaxial cylinders of the diameter D at potentials $V_1 = V_3 = -100$ V, and $V_2 = 0$ V, with equal lengths L. The

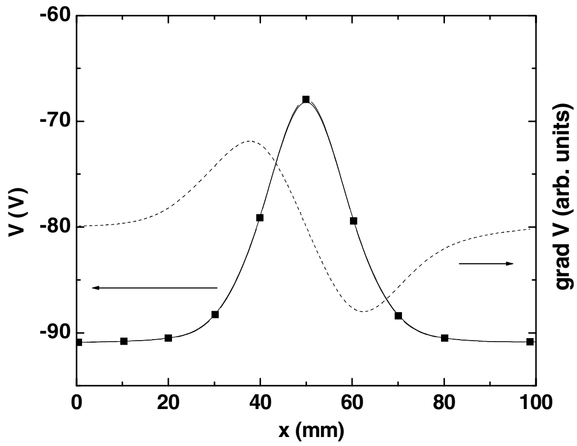

Figure 4. The variation of the potential and its derivative along the optical axis as a function of the *x* position for a three-element einzel lens.

focal properties are studied by sending a beam of particles parallel to the optical axis and determining the focal length as a function of the beam energy.

Figure 2 shows a simulation of electron trajectories through an einzel lens. The object is simulated by a monoenergetic punctual source. The angle that the initial electron velocities make with the optical axis varies from $-10°$ to $+10°$.

Figure 3 shows *P*–*Q* curves for many beam energies obtained by varying both the source position along the optical axis and/or the beam energy for an einzel lens. The symbols represent results of the SIMION calculations and the solid curves are fits of equation (2).

The determination of the potential function along the optical axis is of central importance in order to study the properties of an electrostatic lens. The axial potential distribution $V(x)$ and its gradient $\mathrm{d}V(x)/\mathrm{d}x$ are shown in figure 4.

Summary

We have described some computer-based activities to bring the study of charged particle optics to undergraduate students, to be performed as a part of a one-semester accelerator-based experimental course [10]. The computational simulations were carried out using the commercially available SIMION program, which addresses interactive methods for simulating a broad variety of general ion optics jobs. Although no attempts have been made in this experimental course to approve the simulation results, the simulations help the students in better understanding of the accelerator optical elements. The performance parameters, such as focal length, are obtained. The three-electrode einzel lens is exemplified here as a study case.

Acknowledgment

This work is supported in part by CNPq and FAPERJ (Brazil).

References

[1] Busch H 1927 Über die Wirkungsweise der Konzentrierungsspule bei der Braunschen Röhre *Arch. Elektrotech.* **18** 583–94 Jena, Physikalisches Instituts

[2] Ruska E 1980 The early development of electron lenses and electron microscopy *Seiten* (Stuttgart: Hirzel Verlag) p 140
[3] Hawkes P 2004 Recent advances in electron optics and electron microscopy *Annales de la Fondation Louis de Broglie* vol 29 (Hors série 1) pp 837–55
[4] Dahl D A 1995 SIMION 3D version 7.0 User's Manual
[5] Heddle D W O 2000 *Electrostatic Lens Systems* 2nd edn (London: IOP Press)
[6] C P O Programs http://cpo.ph.man.ac.uk/
[7] Cubric D, Lencová B, Read F H and Zlamal J 1999 *Nucl. Instrum. Methods A* **427** 357
[8] Moore J H, Davis C C and Coplan M A 1989 *Building Scientific Apparatus* 2nd edn (Reading, MA: Addison Wesley)
[9] Sise O, Ulu M and Dogan M 2005 *Nucl. Instrum. Methods Phys. Res. A* **554** 114–31
[10] Santos A C F, Magalhães S D and de Castro Faria N V 2007 *Nucl. Instrum. Methods Phys. Res. B* **261** 264–7